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U.S. AERONAUTICS AND SPACE ACTIVITIES

JANUARY 1 TO DECEMBER 31, 1959

MESSAGE

FROM

THE PRESIDENT OF THE UNITED STATES

TRANSMITTING

THE SECOND ANNUAL REPORT ON THE NATION'S ACTIVITIES IN THE FIELDS OF AERONAUTICS AND SPACE, PURSUANT TO SECTION 206(b) OF THE NATIONAL AERONAUTICS AND SPACE ACT OF 1958

FEBRUARY 25, 1960.—Referred to the Committee on Science and Astronautics and ordered to be printed with illustrations

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Military space activities

The Department of Defense placed special emphasis on its Discoverer satellite program in 1959. Its objective: the testing of components, propulsion and guidance systems, and techniques for several U.S. space projects. A capsule recovery operation, so far unsuccessful, is a principal technique being tested in the program.

Eight of the nine military space launchings were of Discoverer vehicles (six attained orbit): the ninth, in the Transit navigation satellite program, failed to achieve orbit but yielded useful operational data.

Also included among key Defense Department projects were the following: Project Argus, in which the effects of nuclear explosions in the exosphere are being studied; Project Notus, a communications satellite system for long-range radio communication; Project Shepherd, a combined Minitrack-Doploc fence to serve as a space surveillance system for defense purposes; and Project Longsight, a research study concerned with military needs in space technology.

Shepherd

In February 1959, elements of the active Minitrack fence became operational on a 24-hour basis. This system functions to detect, indetify, and predict orbits of nonradiating objects in space. The objective of this program is to obtain at the earliest practicable date a space surveillance tracking system capable of satisfying military and other requirements.

The first definite occurrence of passive detection took place on January 22, 1959, on Sputnik III, at the Forest City station. Since that time, the satellites which have been repeatedly detected by this system include Discoverer, Vanguard, and Lunik vehicles.

In 1959, decision was reached to reorient the effort of this project toward improved second-generation tracking systems. Plans were formulated for phasing out some aspects of the interim "fence" and for implementing the reoriented program. These plans include the closing down of some interim stations and increasing the operational capabilities of other stations.

http://www.au.af.mil/au/awc/space/books/spires/ch02.pdf

Beyond Horizons
A Half Century of Air Force Space Leadership
Revised Edition
David N. Spires

Senior Editor
George W. Bradley III
Associate Editors
Rick W. Sturdevant
Richard S . Eckert

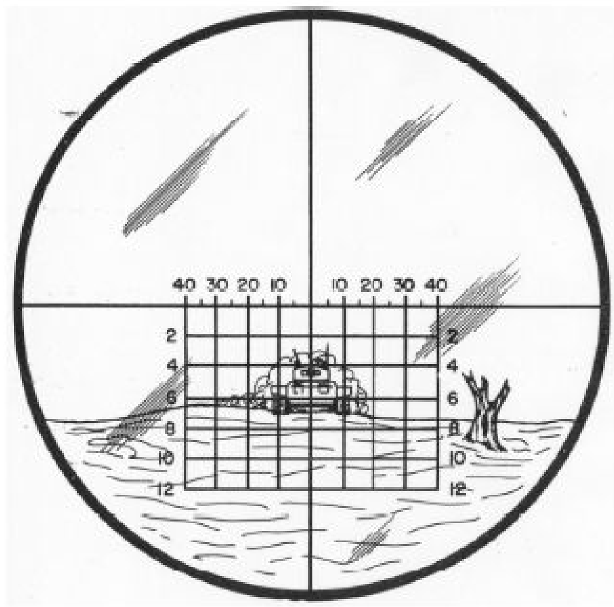
Air Force Space Command
in association with
Air University Press
1998

In addition to its own Samos and MIDAS satellite projects, under ARPA's direction the Air Force provided launch support to the Navy's Transit navigational satellite, designed to support Polaris submarines, and the Army's Notus communications satellite effort ." The most important, however, proved to be the growing detection, tracking and satellite cataloguing project known as the Space Detection and Tracking System (SPADATS) . Begun hurriedly under the name Project Shepherd by ARPA in response to Sputnik, all three services were to participate . The Air Force, under Project Harvest Moon (later Spacetrack), would provide the Interim National Space Surveillance and Control Center (INSSCC) data filtering and cataloguing center at its Cambridge Research Center in Massachusetts . Early efforts brought together radar data from MIT's Lincoln Laboratory's Millstone Hill radar at Westford, the Stanford Research Institute in Palo Alto, California, and an ARDC test radar at Laredo, Texas. Sensors included the new Smithsonian Astrophysical Observatory's Baker-Nunn satellite tracking cameras that it procured for tracking the IGY satellites and available observatory telescopes. The Air Force would also devise the development plan for the future operational system .

ARPA assigned the Navy responsibility for developing and operating its east-west Minitrack radar fence and its data processing facility in Dahlgren, Virginia. Originally designed to support Project Vanguard, the Navy redesignated its sensor and control operation Spasur (Space Surveillance) . The Army portion, termed Doploc, envisioned a doppler radar network to augment Spasur and, together, feed data to the INSSCC for cataloguing, trajectory prediction, and dissemination . ARPA and the three services realized the system's limited capability, but agreement on funding necessary improvements proved difficult to achieve . After the Army dropped out of the picture, the Air Force and Navy contested for operational control of the system. The Navy seemed to prefer operating a separate system, while the Air Force wanted its Air Defense Command (ADC) to acquire management responsibility and NORAD to possess operational control. By mid-1959, the controversy had reached the Joint Chiefs of Staff, where it became embroiled in a major roles and missions contest among the services .

http://sill-www.army.mil/famag/1961/JUN_1961/JUN_1961_FULL_EDITION.pdf

ARTILLERY TRENDS



U S Army Artillery and Missile School

June 1961

SATELLITE DETECTOR SCREEN PLANNED

The Army Ordnance Ballistics Research Laboratories at Aberdeen Proving Grounds, Maryland, are reported to have the high spot in considerations for a satellite-detection screen in the United States. The screen is especially qualified for detection of "dark" or nontransmitting satellites with the DOPLOC system.

The present single test facility with a radar transmitter at Fort Sill, Oklahoma, and "detector" at Forrest City, Arkansas, first located and identified the "mystery satellite" that proved to be part of Discoverer V. The key to the system is the complex of high-speed computers at BRL to analyze the data transmitted from Forrest City.

Tracking in Space by DOPLOC*

L. G. DEBEY†

Summary—**A satellite and space vehicle tracking system of the Doppler type, known as DOPLOC, is described. The characteristics of the heart of the system, a phase-locked tracking filter, are discussed from the viewpoints of bandwidth, signal-to-noise ratio, and accuracy of Doppler frequency measurement. System sensitivity to low energy signals is shown to be 2×10^{-20} watts at a bandwidth of 1 cps. Tracking ranges vs frequency are given for constant gain and constant aperture antennas. The advantages of DOPLOC in satellite tracking programs are briefly discussed.**

INTRODUCTION

A NUMBER of acronyms have made their appearance in the technical literature over a period of several years. DOPLOC, derived from DOPpler phase LOCk, is but one of many such names coined to permit quick reference to a particular system of equipment. It is a type of radiometric tracking system designed particularly for obtaining trajectory or orbit information from missiles, satellites, or space vehicles in flight. As its name implies, it is based on the use of Doppler techniques enhanced by correlation detection methods which employ narrow-band, electronic, frequency tracking filters. The purpose of this paper is to describe briefly the DOPLOC system and to discuss some of its more important advantages in detecting and tracking low energy level signals received from satellites and space vehicles.

Instrumentation systems based on the Doppler principle have been in use for many years, primarily as precision tracking devices at the various guided missile test ranges. The first such system to attain wide application was the Doppler Velocity and Position (DOVAP) system developed by the Army Ordnance's Ballistic Research Laboratories in 1945 for use at White Sands Missile Range. Doppler systems of the DOVAP type are beacon systems; *i.e.*, a missile-borne receiver-transmitter is used to amplify and retransmit a CW signal originating at a ground transmitting station. The chief advantage of such a system lies in the fact that the frequency and wavelength of the radiation are accurately known.

THE DOPLOC SYSTEM

Doppler systems, like radars, may also be used without cooperative beacons in the satellite. With beacons, signal levels are relatively high, whereas without beacons energy must be reflected from the satellite, requiring either high power illuminating transmitters or greatly reduced tracking range. Because of the high sensitivity of the DOPLOC system, it may be used advantageously in reflection Doppler applications. For the purposes of this paper, however, its use with a beacon will be described.

The vehicle to be tracked is assumed to carry a small radio transmitter. The frequency of the signal from the transmitter, as received at a DOPLOC ground station, is compared with a stable local oscillator (STALO) and the change in frequency vs time (Doppler shift) is measured and recorded. Fig. 1 shows a simplified block diagram of a typical DOPLOC receiving system. All components are commercially available items which have conventional characteristics except the phase-locked tracking filter. The tracking filter is the heart of the system and provides the special characteristics which permit detection of very low energy signals. It has been described by Richard.[1] Some of its more important characteristics will be reviewed briefly here as a prelude to a discussion of its application to the space tracking problem.

THE TRACKING FILTER

The principle of operation of the tracking filter is shown in Fig. 2. The phase of the input signal is compared with that of a voltage-controlled oscillator (VCO), an error voltage is developed, and, after filtering in an equalizer network, is used to control the frequency and phase of the VCO. This electronic servo system has been designed to yield essentially zero tracking error for constant rate of change of frequency. The small errors which do exist are due to inability to synthesize perfectly the theoretically correct third-order transfer function on which the design of the filter was based.

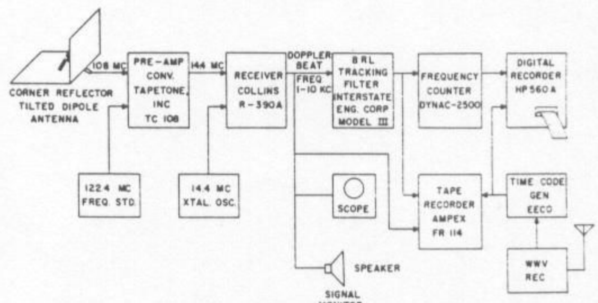


Fig. 1—108-mc DOPLOC station. Basic Doppler satellite tracking station using BRL tracking filter.

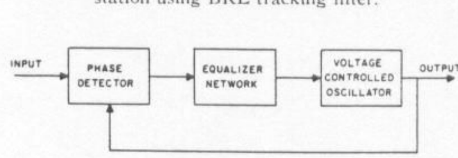


Fig. 2—Basic tracking filter system.

* Manuscript received by the PGMIL, February 1, 1960.
† Ballistic Res. Labs., Aberdeen Proving Ground, Md.

[1] V. W. Richard, "The DOPLOC Tracking Filter," Ballistic Res. Lab., Aberdeen Proving Ground, Md., BRL Rept. No. 1173; October, 1958.

The response of the filter to a step function of rate of change of frequency $\dot{f}$ from 0 to 5 cps is shown in Fig. 3 for a nominal filter bandwidth of 5 cps. Note that after the initial transient of error signal the latter drops to a value corresponding to a tracking phase error of about five degrees. It has been shown by Katz and Honey[2] that the bandwidth of the filter may be determined from

$$B = 1.38\sqrt{\frac{\dot{f}}{E}} \qquad (1)$$

where B = bandwidth in cps, E = peak phase transient in radians, and $\dot{f}$ = amplitude of $\dot{f}$ step function in cps. Thus

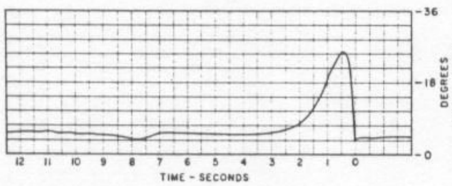


Fig. 3—Measured transient phase error vs time.

from Fig. 3 the actual bandwidth may be calculated and found to be 4.4 cps. Final calibration of the filter is accomplished by this method.

The signal-to-noise improvement is a function of the ratio of tracking filter bandwidth to input bandwidth. Amplitude noise at the input to the filter is converted to phase noise at the output of the filter. Katz and Honey[2] have shown that

$$\left(\frac{N}{S}\right)_0 = \sqrt{2}\,\theta_0$$

where θ_0 = rms phase jitter in output signal expressed in radians. For a given maximum output phase jitter, the maximum tolerable noise-to-signal ratio at the input to the filter will be

$$\left(\frac{N}{S}\right)_i = \sqrt{2}\,\theta_0\sqrt{\frac{B_i}{B_0}}\,.$$

It has been found by experiment that $\theta_0 = \frac{1}{2}$ radian is the maximum jitter that will permit essentially continuous tracking without an occasional peak phase excursion causing the filter to drop lock, and corresponds to a 6-db output signal-to-noise ratio. Fig. 4 relates $(N/S)_i$ to filter bandwidth for $\theta_0 = 1, \frac{1}{2}$, and $\frac{1}{4}$ radians.

Of the several sources of radio noise, the two of greatest importance in space tracking are internal receiver noise and cosmic noise. It is assumed that man-made noise can be held within tolerable limits through careful selection of receiving sites and good engineering practices.

[2] L. Katz and R. Honey, "Phase-lock Tracking Filter," Interstate Engineering Co., Anaheim, Calif., Final Engrg. Rept. for the Ballistic Res. Labs.; May 22, 1957.

In a Doppler system the Doppler frequency f_d and rate of change of frequency $\dot{f}_d$ is proportional to the carrier frequency f_c. Eq. (1) shows that the bandwidth required in the filter to maintain lock is proportional to the square root of $\dot{f}_d$. The output noise power will therefore increase as the square root of the carrier frequency assuming a constant receiver noise figure. The noise at the receiver output due to galactic sources varies as $f^{-1.8}$. Fig. 5 shows the internal receiver and galactic noise magnitudes as functions of frequency referred to the receiver input. This figure also shows the received signal power required, for output signal-to-noise ratio of 0 db, and two examples of the tracking range capability of the DOPLOC system. The assumed values of the parameters are within the current state of the art.

The first example assumes a fixed and relatively low antenna gain. For low altitude satellites, where the angular velocity of the satellite with respect to a receiving site is high, antenna tracking is not always practical and nearly omnidirectional antennas are desirable. If narrow beam antennas are used in such a way that the duration of received signal is a function of beamwidth, as in search or acquisition operations, the maximum gain will be a func-

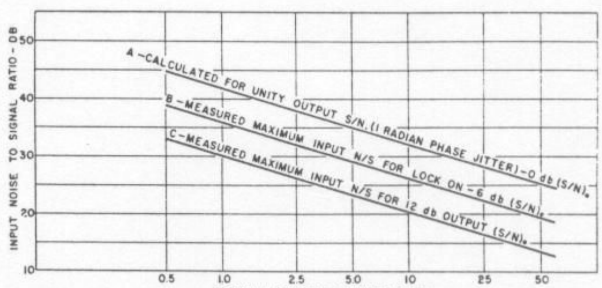


Fig. 4—Signal-to-noise improvement by BRL tracking filter. Tracking filter bandwidth vs input noise-to-signal ratio.

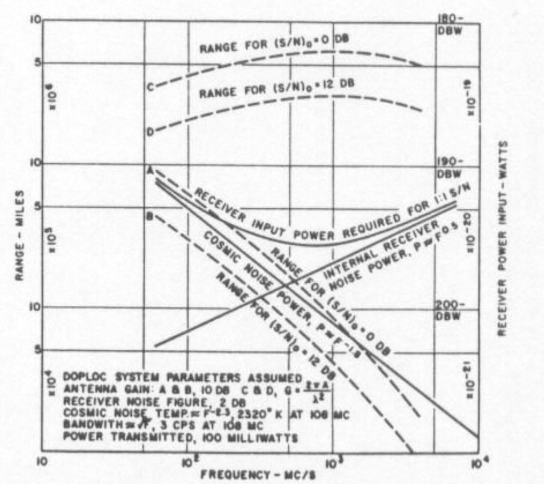


Fig. 5—Noise, receiver signal input power, and tracking range vs frequency.

tion of minimum beamwidth. Since the beamwidth determines the duration of signal, which in turn fixes the integration time available for determining frequency, there will be a lower limit of beamwidth which will yield the desired accuracy of frequency measurement as will be shown later [see (6)].

In the second example, a constant antenna aperture is assumed and the gain increases with frequency. Such antennas are applicable to the space probe problem where angular velocities are very low over long intervals of time and very narrow beamwidth antennas may be tracked successfully. The gain in such cases is limited only by the tolerable physical size of the antenna structure. A parabolic antenna with a diameter of 100 feet has been assumed for this case.

From Fig. 5 it is apparent that Doppler systems having antennas with fixed maximum gains should be operated at the lowest frequency compatible with the tolerable errors caused by the ionosphere if maximum tracking range is to be achieved. On the other hand, a system with antennas of fixed maximum size will realize maximum tracking range with a frequency near 900 mc.

Frequency Coverage

The frequencies that have been used for space-earth communication and tracking have ranged from about 20 mc to 1000 mc. In general, each different satellite or space probe project makes use of a different frequency. Unlike other frequency or phase-locked systems, in which the entire receiver is in the feedback loop and hence must be carefully adjusted to optimize the transfer function for each significant change in frequency, the DOPLOC system maintains its basic characteristics throughout the 55- to 970-mc range for which equipment is currently available.

Automatic Lock-On

Typical Doppler frequencies resulting from satellite tracking may range from 2 to 14 kc for a carrier frequency of 108 mc. Special techniques are required to search this frequency range, detect the signal buried in noise, adjust the tracking filter VCO to the signal frequency, and lock it in phase with the signal. For slowly varying Doppler frequencies these operations may be accomplished by a human operator. For high rates of change of frequency, rapid, automatic, methods are desirable. A comb filter frequency search device has been developed and used successfully to perform these operations. Selected narrow frequency bands (1 to 50 cps) in the 2- to 14-kc band are tested periodically for presence of signal, and when signal is observed in any such narrow band the tracking filter VCO is shifted electronically to this frequency and locked to it. For the model currently in use, 120 bands are scanned every 1.2 seconds. Most Doppler signals can be acquired in a few scan cycles. A newer automatic lock-on system of this same general type has been developed to permit scanning 1200 bands in the 2- to 14-kc range in 0.1 second, thus assuring automatic lock-on to any signal which the tracking filter can track.

Frequency Measurement

In a Doppler system the basic measurement is one of frequency, and the accuracy to which frequency can be measured is a direct indication of the figure of merit of the system. The error to be expected in frequency measurements using the tracking filter may be derived from the expression previously given for output phase jitter. The input and output noise to signal voltage ratios are related by

$$\left(\frac{N}{S}\right)_i = \left(\frac{N}{S}\right)_0 \sqrt{\frac{B_i}{B_0}}\,. \tag{3}$$

Substituting in (2),

$$\theta_0 = \frac{1}{\sqrt{2}\left(\frac{S}{N}\right)_0}\,. \tag{4}$$

θ_0 is the rms value of the phase noise on the output of the filter and constitutes the error that will be made in measuring phase at the beginning and at the end of any interval T_i over which the frequency f is to be averaged. If the total change of phase of the signal during the interval T_i is $\phi = 2\pi f\, T_i$, then the rms error in frequency measurement will be

$$\Delta f = \frac{\sqrt{2}\,\theta_0}{\phi} \times f. \tag{5}$$

Substituting for θ and ϕ

$$\Delta f = \frac{\sqrt{2}}{\sqrt{2}\left(\frac{S}{N}\right)_0} \times \frac{1}{2\pi f T_i} \times f = \frac{1}{2\pi T_i\left(\frac{S}{N}\right)_0}\,. \tag{6}$$

For a typical integration time of 0.5 second and a 12-db output signal-to-noise ratio, Δf is 0.08 cps. This is the rms value of uncertainty of frequency measurement.

Experimental tests have been conducted in an attempt to confirm the validity of (6). The standard deviations of Δf for tracking filter input noise-to-signal ratios of 0, 6, 12, and 24 db were determined and the experimental value of the constant in (6) was derived. The average value of the constant for the four signal-to-noise ratios was found to be 0.157 as compared to the theoretical value of $\frac{1}{2}\pi =$ 0.159, a discrepancy of approximately one per cent. The values quoted are for the case where $f = 10$ kc, $B = 10$ cps, and $T_i = 0.1$ second. Further tests will be required to evaluate completely the frequency measuring capability of the DOPLOC system, but from these preliminary results it appears that frequency can be measured to within very close limits of theoretically attainable values.

DOPLOC Results

The results of hundreds of tracking operations at a frequency of 108 mc have shown that the peak random scatter of individual frequency measurements is about 2.5 cps. In view of the demonstrated ability of the receiving and frequency measuring equipment to measure the frequncy of noisy signals with a standard deviation of frequency error considerably smaller than the observed scatter, it appears that the conclusion must be drawn that the observed scatter in frequency is due to perturbations introduced by the propagating medium. Inhomogeneity of the ionosphere is most likely the biggest contributing factor, although antenna phase perturbations caused by gyrations of the satellite in flight also contribute to the total phase noise.

The DOPLOC system has been used extensively in space tracking operations. Starting with the first United States satellite, it has provided precise information on the velocity history of selected portions of the trajectories of all but a very few of the satellites and space probes that have been launched. The inherent high sensitivity of the DOPLOC system to signals of very low energy (2×10^{-20} watts, -197 dbw or 0.001 μv across 50 ohms for 1-cps bandwidth) has permitted the use of conventional low gain, wide coverage antennas to achieve horizon to horizon tracking at ranges in excess of 25,000 miles. The system has been found to track properly to within a few decibels of theoretical limits, a characteristic due in large part to the careful design and high stability of the phase-locked tracking filter. Operationally, it has been found to be practical to change bandwidths, over the selectable range of 1 to 50 cps, in accordance with the information content of the signal, and thus to achieve maximum signal-to-noise ratios.

DOPLOC data have been used in a number of ways. Digital data obtained in real time during the launch phase have been transmitted via double parity check data transmission systems to computing centers for use in establishing the orbit injection parameters. Data from passes of a satellite within line-of-sight of the stations have been similarly used by computing centers for improving the orbit parameters previously determined. More recently, Doppler data from a single pass of a satellite over two stations have been used to determine completely the orbit parameters. The analytical methods employed in obtaining the orbit are reported by Patton.[3] Patton's method will also yield orbit solutions with data taken from a single Doppler station, and tests of the method using simulated error-free data have demonstrated the feasibility of establishing orbit parameters after only a few minutes of high-speed computer time. The key to successful determination of orbits with real data lies in obtaining data with small values of random and systematic error. Of the injection parameters which define a particular orbit, the velocity parameters are of greatest importance, particularly when only a small segment of the orbit is subject to measurement. Doppler systems thus have distinct advantages over systems which measure position and derive velocity component by differentiating independent position measurements. The DOPLOC system, because of its high resolution and accuracy in velocity measurement, ranks among the best of available satellite trajectory determining systems, and because of its narrow band characteristics, potentially it has the greatest tracking range of any system now in use or contemplated, assuming the same characteristics for other systems.

Over the span of time since the first guided missile was fired in this country, there have been often-repeated predictions that one or more of the more sophisticated radars, interferometers or other tracking systems would replace Doppler systems in precision tracking applications. Far more sophisticated and complex systems now have been developed, and second-generation systems are in the research and development stage. In spite of the technological advancements in these areas, Doppler systems continue to be the choice of many missile system designers for obtaining the most needed information concerning flight performance. There is no indication that contemplated developments in these other fields will replace the Doppler systems which will grow out of systems of the current DOPLOC type.

[3] R. B. Patton, Jr., "Orbit determination from single pass Doppler observations," this issue, p. 336.

Orbit Determination from Single Pass Doppler Observations*

R. B. PATTON, JR.†

Summary—**This paper presents a method for determining the orbit of a satellite by observing, in the course of a single pass, the Doppler shift in the frequency of a CW signal transmitted from the ground and reflected by the satellite to one or more ground-based receivers at remote sites. The method is sufficiently general that, with minor modification, it may be applied to any type of satellite or ICBM tracking measurements. The computation consists of improving approximations for initial position and velocity components by successive differential corrections which are obtained from a least squares treatment of an over-determined system of condition equations while imposing elliptic motion as a constraint. Methods for obtaining approximations for the initial position and velocity components are likewise discussed. Results are presented for computations with typical input data.**

INTRODUCTION

BY observing the Doppler shift in a CW signal, an orbit may be determined from a few minutes of data recorded in the course of a single pass of a satellite. It is immaterial whether the source of the signal is an air-borne transmitter in the satellite or ground-based instrumentation. However, while the latter requires a more complex over-all instrumentation system, it both simplifies and increases the reliability of the computing methods which are involved in the data reduction process. The method of solution, which is presented herein, has been developed for a system with a ground-based transmitter, but it may be readily applied to observations from a system in which the signal source is carried in the satellite. Indeed, with minor modification, the method may be applied to any type of satellite or ICBM tracking measurements, since it consists essentially of standard least squares and differential correction techniques.

Inasmuch as it was desired to develop a rapid, as well as reliable, method of determining the orbit of a satellite tracked by a Doppler system employing a minimum of instrumentation, emphasis was initially directed toward the development of a solution for a set of data recorded at a single receiver during a single pass of the satellite. To strengthen the geometry of the system, and therefore the solution, the single receiver requirement was relaxed, but the limitation of single pass measurements was considered essential. This results in a relatively short time interval of observation which in turn permits several simplifying assumptions. First, while the Earth is treated geometrically as an ellipsoid, dynamically it is considered to be spherical. In addition, drag as well as ionospheric and atmospheric refraction may be neglected without serious loss in accuracy. This reduces the problem to one of determining the six parameters of a Keplerian orbit from data recorded by a Doppler system with a ground-based transmitter. Observations from such a system will be referred to in this paper as passive data, as opposed to active data which would be obtained from a system that included an air-borne transmitter as an integral part of the tracking instrumentation. From a data reduction point of view, there are several distinct advantages in having the transmitter on the Earth's surface. Of primary interest are the following.

1) Both the transmitter frequency and drift may be accurately measured if the signal source is ground-based whereas they become additional unknowns for any system that includes an air-borne transmitter. Moreover, they are weakly determined when treated as unknowns in the latter system, and in turn, they seriously degrade the accuracy with which the orbital parameters are determined.
2) Measurements by a single receiver are insufficient for a reliable solution when the transmitter is carried by the satellite. On the other hand, single receiver solutions are frequently possible with passive data so that reductions may still be obtained even when several receiving sites of a multi-receiver system may fail to record data.
3) Increased reliability is obtained through the complete elimination of air-borne equipment.

DESCRIPTION OF OBSERVED DATA

In order to arrive at a computing method, it was necessary to consider the various forms in which the data may be recorded. The observations consist of either Doppler frequency or the Doppler period, which may be readily converted to frequency. In the sense in which it is used here, Doppler frequency is defined to be the frequency obtained by beating a local oscillator against the received signal reflected from the satellite and then correcting the output for the bias introduced by the difference in frequency between the transmitter and the local oscillator. If the Doppler frequency is plotted as a function of time, one obtains a curve of the form plotted in Fig. 1(a), usually referred to as an S curve. The asymmetry is typical for a system in which the ground-based transmitter and receiver are separated by an appreciable distance and the orbit is not symmetrical with respect to the base line joining the two instrumentation sites. If observations are recorded at

* Manuscript received by the PGMIL, February 1, 1960.

† U. S. Army Ballistic Res. Labs., Aberdeen Proving Ground, Md.

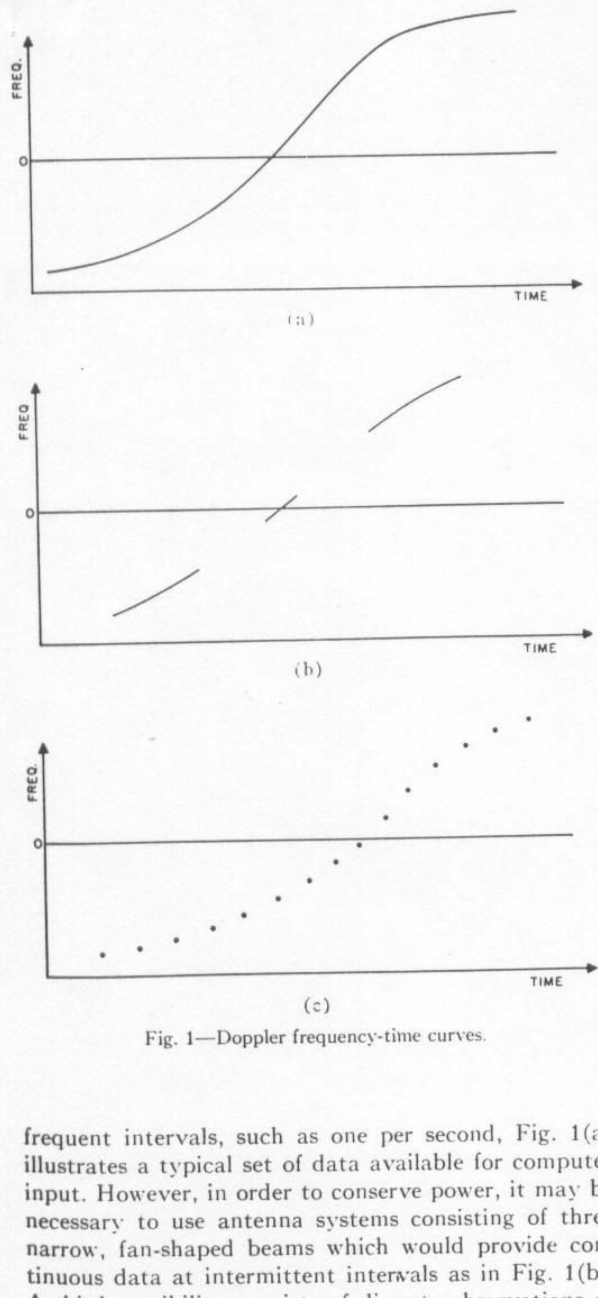


Fig. 1—Doppler frequency-time curves.

frequent intervals, such as one per second, Fig. 1(a) illustrates a typical set of data available for computer input. However, in order to conserve power, it may be necessary to use antenna systems consisting of three narrow, fan-shaped beams which would provide continuous data at intermittent intervals as in Fig. 1(b). A third possibility consists of discrete observations at regular intervals as shown in Fig. 1(c). Such data could be obtained by a system which scanned the sky with a narrow, fan-shaped beam.

Any of these sets of data may be used readily as input for the computing procedure. Whenever possible, this input consists of the total cycle count rather than the Doppler frequency, *i.e.*, the area under the curves or arcs of curves presented in Fig. 1(a) and 1(b). Hence, this method of solution is based, in a sense, upon every available observation rather than upon a selected few of the total observations (which would be the case if the computing input were limited to a representative number of frequency measurements). Experience has shown this to yield a very significant gain with regard to the accuracy and convergent properties of the solution.

The Solution

The method of solution consists of fitting a computed curve to a set of Doppler observations by a least squares technique in which a compatible set of approximations to the six orbital parameters are derived and then improved by repeatedly applying differential corrections until convergence is achieved. Keplerian motion is assumed. The equations of condition are derived from the Taylor expansion about that point which consists of the approximate values of the six orbital parameters. All second and higher order terms of the expansion are neglected. Convergence is greatly dependent upon the set of parameters which are selected to describe the orbit. Not every set of orbital parameters will yield normal equations which are sufficiently independent to permit a solution with this method. It has been found that parameters consisting of position and velocity components for a given time readily yield a convergent solution, whereas a set comprising the semi-major axis, eccentricity, mean anomaly at epoch, inclination, right ascension of the ascending node, and argument of perigee proved to be altogether hopeless.

As stated previously, the observations will consist either of Doppler frequency as a function of time, or period measurements which may be converted to Doppler frequency. The desired computer input, on the other hand, is the total cycle count. When a continuous record of observations is recorded for a substantial period of time, the total cycle count may be obtained by the simple process of summation over time intervals that, in general, will be of several seconds duration. If λ is the wavelength of the radiated signal and N_{ij} the total number of Doppler cycles resulting at the ith receiver from the motion of the satellite between times t_j and t_{j+1}, it follows that $v_{ij} \equiv \lambda N_{ij}$ is a measure of the total change in path length from the transmitter to the satellite to the ith receiver between times t_j and t_{j+1}. Let g_{ij} be defined as the actual change in path length. It follows from Fig. 2 that:

$$g_{ij} = (TS_{j+1} + R_iS_{j+1}) - (TS_j + R_iS_j), \qquad (1)$$

where T is the transmitting site, R_i the location of the ith receiver, S_{j+1} the position of the satellite at time t_{j+1}, and S_j the position of the satellite at time t_j. In the event that the observations consist only of discrete measurements of frequency, the same definitions will apply, but the time interval from t_j to t_{j+1} will be limited to one second.

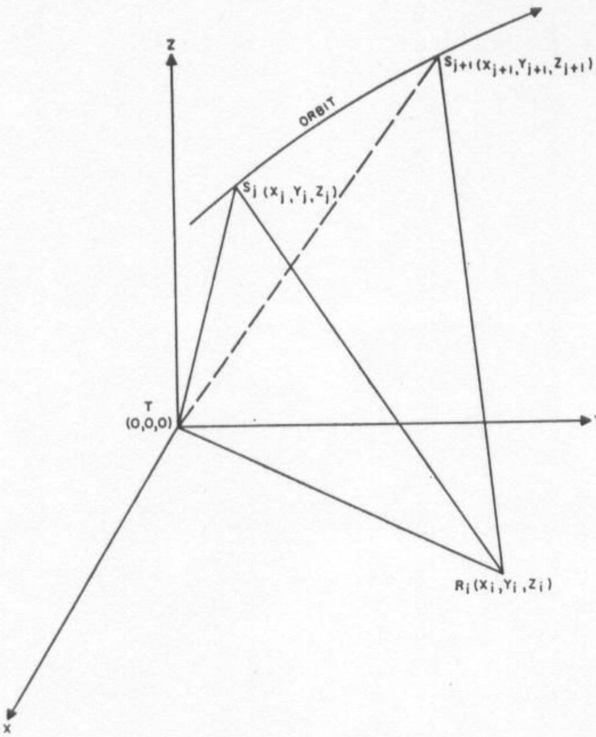


Fig. 2—Problem geometry.

The solution consists of improving a set of position and velocity components which have been approximated for a specific time. The latter will be defined as t_0, and as a matter of convenience, it is generally taken as the time at which tracking is initiated. Although the location of the coordinate system in which the position and velocity vectors are defined is relatively immaterial from the standpoint of convergence, a system fixed with respect to the Earth's surface does provide two distinct advantages. First, the function g_{ij} is somewhat simplified since the coordinates of the instrumentation sites remain fixed with respect to time. Of greater significance, however, is the fact that the method may be altered to accept other types of measurements for input by merely changing the definition of the function g_{ij} and correspondingly, the expressions for its derivatives, with no additional modification to the balance of the procedure which in fact, constitutes the major portion of the computing process.

The set of initial approximations for the position and velocity components are defined for time t_0 as $(x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0)$. The reference frame is the xyz-coordinate system which is defined in the following section. If second and higher order terms are omitted from the Taylor expansion about the point $(x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0)$, the equations of condition may be written in matrix form,

$$\Delta V = J\Delta X, \tag{2}$$

where

$$J \equiv \left(\frac{\partial g_{ij}}{\partial x_0}, \frac{\partial g_{ij}}{\partial y_0}, \frac{\partial g_{ij}}{\partial z_0}, \frac{\partial g_{ij}}{\partial \dot{x}_0}, \frac{\partial g_{ij}}{\partial \dot{y}_0}, \frac{\partial g_{ij}}{\partial \dot{z}_0} \right),$$

$$\Delta V \equiv (\Delta v_{ij}) = (v_{ij} - g_{ij}), \tag{3}$$

$$\Delta X \equiv \begin{pmatrix} \Delta x_0 \\ \Delta y_0 \\ \Delta z_0 \\ \Delta \dot{x}_0 \\ \Delta \dot{y}_0 \\ \Delta \dot{z}_0 \end{pmatrix},$$

for all values of i and j. Hence, J is a matrix of order $(i \cdot j \times 6)$, ΔV a matrix of order $(i \cdot j \times 1)$, and ΔX a matrix of order (6×1). Since there are six unknowns, a minimum of six equations are required for a solution. In practice, sufficient data are available to provide an over-determined system, thus permitting the least squares solution,

$$\Delta X = (J^* J)^{-1} J^* \Delta V, \tag{4}$$

where J^* is the transpose of the Jacobian J. Finally, improved values for the initial conditions are obtained from

$$X + \Delta X,$$

where

$$X \equiv \begin{pmatrix} x_0 \\ y_0 \\ z_0 \\ \dot{x}_0 \\ \dot{y}_0 \\ \dot{z}_0 \end{pmatrix}.$$

As a matter of convenience, no subscripts were introduced to indicate iteration; but at this point, the improved values of X are used for the initial point and the process is iterated until convergence is achieved.

Evaluation of ΔV

Since the function g_{ij} cannot be expressed readily in terms of X directly, the evaluation is obtained implicitly. An ephemeris is computed for the assumed values of the position and velocity vectors at time t_0. Then using (1), g_{ij} may be evaluated for all values of i and j.

In the process of this evaluation, it is expedient to use two rectangular coordinate systems in addition to the Earth-bound system whose origin is at the transmitting site. Referring to Fig. 3, these coordinate systems are defined as follows.

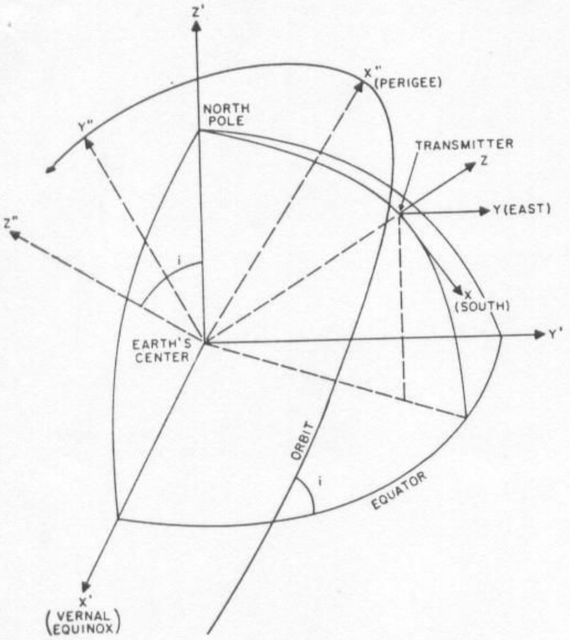


Fig. 3—Coordinate systems.

1) The xyz-coordinate system is a right-hand rectangular system with the origin on the Earth's surface at the transmitter. The x axis is positive south, the y axis is positive east and the z axis is normal to the Earth's surface at the transmitting site.
2) The $x'y'z'$-coordinate system is a right-hand rectangular system with the origin at the Earth's center. The x' axis lies in the plane of the equator and is positive in the direction of the vernal equinox, while the positive z' axis passes through the north pole. The y' axis is chosen so as to complete a right-hand system.
3) The $x''y''z''$-coordinate system is likewise a right-hand rectangular system with the origin at the Earth's center. The $x''y''$ plane lies in the orbital plane of the satellite, with the positive x'' axis in the direction of perigee. The positive z'' axis forms with the positive z' axis an angle equal to the inclination of the orbital plane to the plane of the equator. The y'' axis is chosen so as to complete a right-hand system.

The first step in this phase of the computation consists of computing values for the initial position and velocity components in the $x'y'z'$-coordinate system. This may be achieved by one rotation and one translation which are independent of time, and a second rotation which varies with time. Let the notation $R_i(\alpha)$ indicate the matrix performing a rotation through an angle α about the ith axis of the frame of reference such that the angle is positive when the rotation is in the right-hand direction and i is equal to 1, 2, or 3, according to whether the rotation is about the x, y, or z axis respectively. The desired transformation follows,

$$\begin{pmatrix} x_0' \\ y_0' \\ z_0' \end{pmatrix} = R_3(-\theta_0)R_2(\phi - 90°) \left[\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \begin{pmatrix} \rho_\phi \sin\Delta \\ 0 \\ \rho_\phi \cos\Delta \end{pmatrix} \right], \tag{5}$$

$$\begin{pmatrix} \dot{x}_0' \\ \dot{y}_0' \\ \dot{z}_0' \end{pmatrix} = \dot{R}_3(-\theta_0)R_2(\phi - 90°) \left[\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \begin{pmatrix} \rho_\phi \sin\Delta \\ 0 \\ \rho_\phi \cos\Delta \end{pmatrix} \right] + R_3(-\theta_0)R_2(\phi - 90°) \begin{pmatrix} \dot{x}_0 \\ \dot{y}_0 \\ \dot{z}_0 \end{pmatrix}, \tag{6}$$

where

$\dot{R}_3(-\theta_0) \equiv$ the time derivative of $R_3(-\theta_j)$ when $\theta_j = \theta_0$,
$\theta_j \equiv$ the right ascension of the transmitting site at time t_j,
$\phi \equiv$ the geodetic latitude of the transmitting site,
$\rho_\phi \equiv$ the radius vector from the Earth's center to a point on the Earth's sea level surface at the latitude ϕ,
$\Delta \equiv$ the difference between the geodetic and geocentric latitudes at the transmitting site.

The next step in the computation involves the evaluation of the following orbital parameters:

$a \equiv$ semi-major axis,
$e \equiv$ eccentricity,
$\sigma \equiv$ mean anomaly at epoch,
$i \equiv$ inclination,
$\Omega \equiv$ right ascension of the ascending node,
$\omega \equiv$ argument of perigee.

The evaluation of these orbital parameters is obtained from the following equations:[1]

$$r_0 = \sqrt{(x_0')^2 + (y_0')^2 + (z_0')^2}, \tag{7}$$

$$v_0 = \sqrt{(\dot{x}_0')^2 + (\dot{y}_0')^2 + (\dot{z}_0')^2}, \tag{8}$$

$$r_0\dot{r}_0 = x_0'\dot{x}_0' + y_0'\dot{y}_0' + z_0'\dot{z}_0', \tag{9}$$

$$\mu = gR^2,$$

[1] Derived by Dr. B. Garfinkel, Ballistic Res. Labs., Aberdeen Proving Ground, Md.

where g is the mean gravitational constant and R is the radius of the Earth, which is assumed to be spherical in the development of the equations,

$$a = \frac{\mu r_0}{2\mu - r_0 v_c^2}, \tag{10}$$

$$e = \left| \frac{(r_0 \dot{r}_0)^2}{\mu a} + \left(1 - \frac{r_0}{a}\right)^2 \right|^{1/2}, \tag{11}$$

$$E_0 = \tan^{-1} \left\{ \frac{r_0 \dot{r}_0}{\sqrt{\mu a}\left(1 - \dfrac{r_0}{a}\right)} \right\} + \frac{\pi}{2}\left\{1 - \operatorname{sgn}\left(1 - \frac{r_0}{a}\right)\right\}, \tag{12}$$

where

$$\operatorname{sgn}\left(1 - \frac{r_0}{a}\right) \equiv 1 \quad \text{if} \quad \left(1 - \frac{r_0}{a}\right) > 0,$$

$$\operatorname{sgn}\left(1 - \frac{r_0}{a}\right) \equiv -1 \quad \text{if} \quad \left(1 - \frac{r_0}{a}\right) < 0,$$

$$n = \sqrt{\frac{\mu}{a^3}}, \tag{13}$$

$$\sigma = E_0 - e \sin E_0 - n t_0, \tag{14}$$

$$h_1 = y_0' \dot{z}_0' - z_0' \dot{y}_0', \tag{15}$$

$$h_2 = z_0' \dot{x}_0' - x_0' \dot{z}_0', \tag{16}$$

$$h_3 = x_0' \dot{y}_0' - y_0' \dot{x}_0', \tag{17}$$

$$h = \sqrt{h_1^2 + h_2^2 + h_3^2}, \tag{18}$$

$$k_1 = \frac{h_1}{h}, \tag{19}$$

$$k_2 = \frac{h_2}{h}, \tag{20}$$

$$k_3 = \frac{h_3}{h}, \tag{21}$$

$$T = \sqrt{h_1^2 + h_2^2}, \tag{22}$$

$$N_1 = -\frac{h_2}{T}, \tag{23}$$

$$N_2 = \frac{h_1}{T}, \tag{24}$$

$$i = \cos^{-1} k_3, \quad \text{where} \quad 0 \lesseqgtr i \lesseqgtr \pi, \tag{25}$$

$$\bar{x} = a(\cos E_0 - e), \tag{26}$$

$$\bar{y} = a(\sin E_0)\sqrt{1 - e^2}, \tag{27}$$

$$\boldsymbol{r}_0 \cdot \boldsymbol{N} = x_0' N_1 + y_0' N_2, \tag{28}$$

$$\boldsymbol{r}_0 \times \boldsymbol{k} \cdot \boldsymbol{N} = N_1(y_0' k_3 - z_0' k_2) + N_2(z_0' k_1 - x_0' k_3), \tag{29}$$

$$\omega = (-1)^p \cos^{-1} \left\{ \frac{\bar{x}(\boldsymbol{r}_0 \cdot \boldsymbol{N}) + \bar{y}(\boldsymbol{r}_0 \times \boldsymbol{k} \cdot \boldsymbol{N})}{r_0^2} \right\}, \tag{30}$$

where

$$p = \left\{ \frac{1 - \operatorname{sgn}\left[\bar{x} z_0' + \bar{y}(x_0' k_2 - y_0' k_1)\right]}{2} \right\},$$

$$\Omega = (-1)^q \cos^{-1} N_1, \tag{31}$$

where

$$q = \left\{ \frac{1 - \operatorname{sgn} N_2}{2} \right\}.$$

Having obtained values for a, e, σ, i, Ω, and ω, g_{ij} may be evaluated for each time of observation. The first step in the computing procedure consists of solving for the eccentric anomaly E_j in Kepler's equation,

$$E_j - e \sin E_j = n t_j + \sigma. \tag{32}$$

The position of the satellite as a function of time is determined in the $x''y''z''$-coordinate system.

$$f_j = 2 \tan^{-1}\left[\sqrt{\frac{1 + e}{1 - e}} \tan\left(\frac{E_j}{2}\right)\right]; \tag{33}$$

$$r_j = a(1 - e \cos E_j); \tag{34}$$

$$x_j'' = r_j \cos f_j; \tag{35}$$

$$y_j'' = r_j \sin f_j. \tag{36}$$

z_j'' is zero according to the definition of the $x''y''z''$-coordinate system. A transformation to the $x'y'z'$ coordinates can be achieved by three rotations as follows:

$$\begin{pmatrix} x_j' \\ y_j' \\ z_j' \end{pmatrix} = R_3(-\Omega) R_1(-i) R_3(-\omega) \begin{pmatrix} x_j'' \\ y_j'' \\ 0 \end{pmatrix}. \tag{37}$$

Finally the position in the xyz-coordinate system may be obtained by two additional rotations and a translation.

$$\begin{pmatrix} x_j \\ y_j \\ z_j \end{pmatrix} = R_2(90° - \phi) R_3(\theta_j) \begin{pmatrix} x_j' \\ y_j' \\ z_j' \end{pmatrix} - \begin{pmatrix} \rho_e \sin \Delta \\ 0 \\ \rho_e \cos \Delta \end{pmatrix}. \tag{38}$$

Referring to (1), g_{ij} may be then expressed in terms of the satellite's positions at time t_j and t_{j+1}.

$$\begin{aligned} g_{ij} = {} & \sqrt{x_{j+1}^2 + y_{j+1}^2 + z_{j+1}^2} \\ & + \sqrt{(x_{j+1} - x_i)^2 + (y_{j+1} - y_i)^2 + (z_{j+1} - z_i)^2} \\ & - \sqrt{x_j^2 + y_j^2 + z_j^2} \\ & - \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}, \end{aligned} \tag{39}$$

where (x_i, y_i, z_i) is the surveyed position of the ith receiver. Finally, the residuals Δv_{ij} may be determined from

$$\Delta v_{ij} = v_{ij} - g_{ij}. \tag{40}$$

Evaluation of J

Having evaluated the vector ΔV, there remains the problem of determining the Jacobian J. The necessary differentiation may be carried out numerically, but the computing time will be reduced and the accuracy increased if the derivatives are evaluated from analytical expressions. Recalling that for all values of i and j,

$$J \equiv J\left(\frac{g_{10}, \cdots, g_{ij}, \cdots}{x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0}\right),$$

let

$$J = J_1 J_2 J_3 J_4,$$

where

$$J_1 \equiv \left(\frac{g_{10}, \cdots, g_{ij}, \cdots}{x_{j+1}, y_{j+1}, z_{j+1}, x_j, y_j, z_j}\right), \tag{41}$$

$$J_2 \equiv \left(\frac{x_{j+1}, y_{j+1}, z_{j+1}, x_j, y_j, z_j}{a, e, \sigma, \omega, \Omega, i}\right), \tag{42}$$

$$J_3 \equiv \left(\frac{a, e, \sigma, \omega, \Omega, i}{x_0', y_0', z_0', \dot{x}_0', \dot{y}_0', \dot{z}_0'}\right), \tag{43}$$

$$J_4 \equiv \left(\frac{x_0', y_0', z_0', \dot{x}_0', \dot{y}_0', \dot{z}_0'}{x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0}\right). \tag{44}$$

The orders of the above matrices are $(i \cdot j \times 6)$ for J_1, and (6×6) for J_2, J_3, and J_4. A distinct advantage of this method of evaluating J results from the fact that the function g_{ij} appears only in J_1. Hence, if the solution is applied to other types of measurements, only J_1 needs revision for the appropriate evaluation of J. This is trivial compared to the effort required to derive the derivatives contained in J_2 and J_3. The expressions for many of the elements of these Jacobian matrices are rather long and involved, and therefore will not be presented here, but the complete results are reported in [1].

Initial Approximations

Of primary importance to the success of the method that has been presented, is the capability of establishing a compatible set of initial approximations which are sufficiently close to the actual values to permit convergence of the computing process. It has been determined that the computation will converge with input consisting of data from a single receiver system, if the base line from the transmitter to the receiver is not excessively short, and the initial approximation to $(x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0)$ is moderately accurate. For example, with base lines of the order of 400 miles and either continuous or intermittent observations over time intervals of approximately five minutes, convergence may reasonably be expected when the error in each coordinate of the initial estimate is not in excess of 50 to 75 miles and the velocity components are correct to within ½ to 1 mile per second. For a two-receiver system, on the other hand, an accuracy of 150 to 200 miles in each coordinate and 1 to 2 miles per second in each velocity component has been found sufficient to secure convergence. The computing process has occasionally converged with larger initial errors, but the figures presented are intended to specify limits within which convergence may reasonably be assured.

Clearly, a supporting computation to furnish moderately accurate initial approximations is essential to the successful application of the computing method. Several approaches to this phase of the problem have been considered, but one in particular is preferred. It involves using an analog computer to fit a computed curve to an observed Doppler frequency-time curve. In the fitting procedure, the boundary values of a second-order differential equation are systematically varied until the computed S curve matches the observed S curve. The differential equation has been derived on the basis of a circular orbit and a nonrotating Earth. The initial approximations for the position and velocity components, which serve as input for the primary computation, are then computed from the results obtained by the curve fitting procedure. A variation of this method consists of plotting a frequency-time curve for the observed data and then matching this to the appropriate member of a large family of S curves previously computed for various circular orbits. Both methods have been successfully employed.

The following definitions will be useful in the derivation of the differential equation used in the fitting procedure.

$\boldsymbol{R}_T \equiv$ the radius vector from the Earth's center to the transmitting site.

$\boldsymbol{r}_j \equiv$ the radius vector from the Earth's center to the position of the satellite at time t_j.

$\beta_j \equiv$ the angle between $\boldsymbol{R}_T$ and $\boldsymbol{r}_j$.

$(x_T'', y_T'', z_T'') \equiv$ the position of the transmitting site.

$(x_j'', y_j'', 0) \equiv$ the position of the satellite at time t_j.

$\rho_j \equiv$ the distance from the transmitting site to the satellite at time t_j.

$\rho_{ij} \equiv$ the distance from the ith receiver to the satellite at time t_j.

$H \equiv$ the altitude of the satellite above the Earth's surface.

$v \equiv$ the velocity of the satellite.

To simplify the problem, certain assumptions have been made:

1) the satellite moves in a circular Keplerian orbit,
2) β_j is relatively small throughout the period of observation,
3) the Earth is not rotating.

A number of useful relationships may be derived as a result of these assumptions.

$$|\boldsymbol{r}_j| = r = R + H,$$

where H is constant.

$$v = nr = \sqrt{(\dot{x}_j'')^2 + (\dot{y}_j'')^2}.$$

$$\left(\frac{R}{R+H}\right) = \left(\frac{Rv^2}{\mu}\right).$$

$$\ddot{x}_j'' = -n^2 x_j''; \qquad \ddot{y}_j'' = -n^2 y_j''.$$

$$\cos\beta_j \approx 1.$$

$$\boldsymbol{R}_T = \text{constant}.$$

The reference frame for this derivation is the $x''y''z''$-coordinate system which has been defined previously. It follows from the definition of ρ_j that

$$\rho_j = \sqrt{(x_j'' - x_T'')^2 + (y_j'' - y_T'')^2 + (z_T'')^2}. \quad (45)$$

Differentiating twice with respect to time yields

$$\dot{\rho}_j^2 + \rho_j\ddot{\rho}_j = (\dot{x}_j'')^2 + (\dot{y}_j'')^2 + \ddot{x}_j''(x_j'' - x_T'') + \ddot{y}_j''(y_j'' - y_T''), \quad (46)$$

which may be simplified to

$$\begin{aligned}\dot{\rho}_j^2 + \rho_j\ddot{\rho}_j &= v^2 - n^2(r^2 - x_j''x_T'' - y_j''y_T''),\\ &= n^2(\boldsymbol{R}_T\cdot\boldsymbol{r}_j),\\ &= n^2 rR\cos\beta,\\ &= v^2\left(\frac{R}{R+H}\right)\cos\beta,\\ &= \frac{Rv^4}{\mu}\cos\beta,\\ &\approx \frac{Rv^4}{\mu}.\end{aligned}$$

It follows that

$$\ddot{\rho}_j \approx \frac{\frac{Rv^4}{\mu} - \dot{\rho}_j^2}{\rho_j}. \quad (47)$$

A similar expression may be derived for $\ddot{\rho}_{ij}$. Recalling the definition for g_{ij}, we conclude that

$$\ddot{g}_{ij} \approx \frac{A - \dot{\rho}_j^2}{\rho_j} + \frac{A - \dot{\rho}_{ij}^2}{\rho_{ij}}, \quad (48)$$

where

$$A = \frac{Rv^4}{\mu}.$$

The initial step in the fitting process on the analog computer consists of approximating A, ρ_0, ρ_{i0}, $\dot{\rho}_0$, and $\dot{\rho}_{i0}$, then computing and plotting that curve for $\dot{g}_{ij}$, which satisfies (48) while observing the constraints $\ddot{g}_{i0} = \ddot{v}_{i0}$ and $\dot{g}_{i0} = \dot{v}_{i0}$. The five parameters are adjusted systematically until a sufficiently good fit is obtained. The final value for A may be used to determine the velocity from which the altitude may be estimated on the basis of a circular orbit, but this has not been particularly successful for highly eccentric orbits. The preferred procedure is to assume that $\dot{z}_0 = 0$ and $z_0 = \bar{z}$, where $\bar{z}$ is an estimate of the altitude based on experience. The former is always an adequate approximation while the latter may usually be estimated to within 100 to 200 miles by merely observing the general characteristics of the $\dot{S}$ curve. In the event that the original estimate of z_0 proves to be so poor that it prevents convergence, a series of values for $\bar{z}$ may be tried without becoming involved in excessive computation. Estimates for x_0 and y_0 are obtained by solving the equations

$$\begin{cases} x_0^2 + y_0^2 + \bar{z}^2 = \rho_0^2, \\ (x_0 - x_i)^2 + (y_0 - y_i)^2 + (\bar{z} - z_i)^2 = \rho_{i0}^2. \end{cases} \quad (49)$$

Differentiating ρ_j and ρ_{ij} with respect to time yields, for $j=0$ and $\dot{z}_0 = 0$,

$$\begin{cases} x_0\dot{x}_0 + y_0\dot{y}_0 = \rho_0\dot{\rho}_0, \\ (x_0 - x_i)\dot{x}_0 + (y_0 - y_i)\dot{y}_0 = \rho_{i0}\dot{\rho}_{i0}, \end{cases} \quad (50)$$

which may be solved for $\dot{x}_0$ and $\dot{y}_0$. This completes the computing procedure for the desired set of initial approximations.

Computational Results

Numerous convergent solutions have been obtained with both simulated and actual field data serving as computer input. Since the latter are of major interest, the discussion will be restricted to results obtained from real data. This method of solution was developed specifically for the DOPLOC system, which is reported upon by deBey.[2] The interim version of this Doppler system complex consists of a 50-kw continuous wave illuminator transmitter station located at Fort Sill, Okla., and two DOPLOC receiving stations, one at White Sands Missile Range (WSMR) and a second at Forrest City, Ark. To conserve power, the antenna system has been limited to three narrow, fan-shaped beams which provide continuous data at intermittent intervals as illustrated in Fig. 1(b). The present results were obtained from observations which were recorded during a period when the WSMR receiver was inoperable and, therefore, are derived from data recorded by a single receiver in the course of a single pass over the instrumentation site. Included with the DOPLOC results are orbital parameters which were determined and published by the National Space Surveillance Control Center (Space Track). As an aid in comparing the two sets of determinations, the Space Track parameters have been converted to the epoch times of the DOPLOC reductions.

The initial successful solution with field data from the DOPLOC system was achieved for Revolution 9937 of Sputnik III. Measurements were recorded for 28 seconds in the south antenna beam of the system, 7 seconds in the center beam, and 12 seconds in the north beam,

[2] L. G. deBey, "Tracking in space by DOPLOC," this issue, p. 332.

with two gaps in the data of 75 seconds each. Thus, observations were recorded for a total of 47 seconds within a time interval of 3 minutes and 17 seconds. On the first pass through the computing machine, the computation converged in three iterations to initial position and velocity components that were equivalent to the following orbital parameters:

$$a = 4149 \text{ miles},$$
$$e = 0.0153,$$
$$\sigma = 288.04°,$$
$$i = 65.37°,$$
$$\Omega = 178.24°,$$
$$\omega = 104.62°.$$

For comparison, the orbital parameters reported in Space Track Bulletin No. 230 for 1958 Delta II (Sputnik III) were used to compute the value of the parameters for the same epoch time as that of the solution. The results are as follows:

$$a = 4111 \text{ miles},$$
$$e = 0.0130,$$
$$\sigma = 257.78°,$$
$$i = 65.06°,$$
$$\Omega = 178.22°,$$
$$\omega = 137.95°.$$

In comparing the DOPLOC and Space Track solutions, it will be noted that there is reasonably good agreement in the values for a, e, i, and Ω, particularly for the latter two. This is characteristic of the single-pass solution when the eccentricity is small and the computational input is limited to Doppler frequency. Since the orbit is almost circular, σ and ω are less significant than the other parameters and likewise, are more difficult for either system to determine accurately. However, as a result of the small eccentricity, the sum of ω and σ is a rather good approximation to the angular distance along the orbit from the nodal point to the position of the satellite at epoch time and as such, provides a basis of comparison between the two systems. In the DOPLOC solution, $(\omega+\sigma)=32.66°$ while the Space Track determination yields a value of 35.73°, a difference of 3.07° between the two sets of results. To summarize, when limited to single-pass, single-receiver observations, the DOPLOC system provides an excellent determination of the orientation of the orbital plane, a good determination of the shape of the orbit, and a fair-to-poor determination of the orientation of the ellipse within the orbital plane.

Occasionally, excellent results have been obtained for both σ and ω; but in general, the interim DOPLOC system with its present limitations fails to provide consistently good evaluations of these two quantities. Therefore, in presenting the remaining DOPLOC reductions, σ and ω have been eliminated from further consideration. Results have been indicated in Table I for six revolutions of Discoverer XI, including number 172 which was the last known revolution of this satellite. As a matter of interest, the position determined by the

TABLE I

Comparison of DOPLOC and Space Track Results for Discoverer XI

	Revolution Number	a (miles)	e	i (degrees)	Ω (degrees)	Total Amount of Data (seconds)	Interval of Observation (seconds)
DOPLOC	30	4186	0.0295	80.15	215.92	35	123
Space Track		4189	0.0291	80.01	215.84		
Difference		−3	0.0004	0.14	0.08		
DOPLOC	124	4115	0.0198	80.31	207.50	44	138
Space Track		4135	0.0203	80.10	207.27		
Difference		−20	−0.0005	0.21	0.23		
DOPLOC	140	4138	0.0198	80.44	205.22	55	141
Space Track		4121	0.0178	80.10	205.78		
Difference		17	0.0020	0.34	−0.56		
DOPLOC	156	4143	0.0300	80.79	204.50	25	117
Space Track		4108	0.0148	80.10	204.30		
Difference		35	0.0152	0.69	0.20		
DOPLOC	165	4037	0.0111	79.99	203.63	49	165
Space Track		4099	0.0128	80.10	203.50		
Difference		−62	−0.0017	−0.11	0.13		
DOPLOC	172	4093	0.0189	80.44	203.18	35	90
Space Track		4091	0.0111	80.10	202.84		
Difference		2	0.0078	0.34	0.34		

interim DOPLOC system for this pass indicated an altitude of 82 miles as the satellite crossed the base line 55 miles west of Forrest City. To provide a basis for evaluation of the DOPLOC results, orbital parameters, obtained by converting Space Track determinations to the appropriate epoch times, have been included in the table. Table I also contains a listing of the amount of data available for each reduction in addition to the total time interval within which the observations were collected.

Conclusion

This method of solution has been shown to be both practical and useful by numerous successful applications with real as well as simulated data. Although results have been presented for passive data only, the computing procedure has been altered slightly and applied to active data with considerable success. Computing times are reasonable since convergent solutions have required from 20 to 40 minutes on the ORDVAC with the coding in floating decimal, whereas more modern machines would perform the same computation in 2 to 4 minutes. This method allows the determination of a relatively accurate set of orbital parameters with as little as 1.5 to 3 minutes of intermittent observations from a single receiver when the signal source is a ground-based transmitter. While multi-receiver systems provide numerous distinct advantages, it has been shown that it is possible to obtain, routinely, quite satisfactory results with observations from a single receiver.

It should be emphasized that solutions have been obtained for actual field data, and further that such solutions were completely independent of other measuring systems. Results from the latter were used only for comparison and did not enter any phase of the computations leading to these solutions.

In conclusion, the method is general and therefore, need not be confined to either Doppler observations or Keplerian orbits. For more complex orbits, it may be desirable to replace analytical with numerical differentiation and it will, of course, be necessary to modify the computation for the ephemeris; otherwise the solution will be fundamentally the same. Insofar as the use of other types of observations are concerned , only J_1 and g_{ij} need be modified to allow the method to be applied to observations from any satellite or ICBM tracking system, provided the limitation of Keplerian orbits is retained.

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